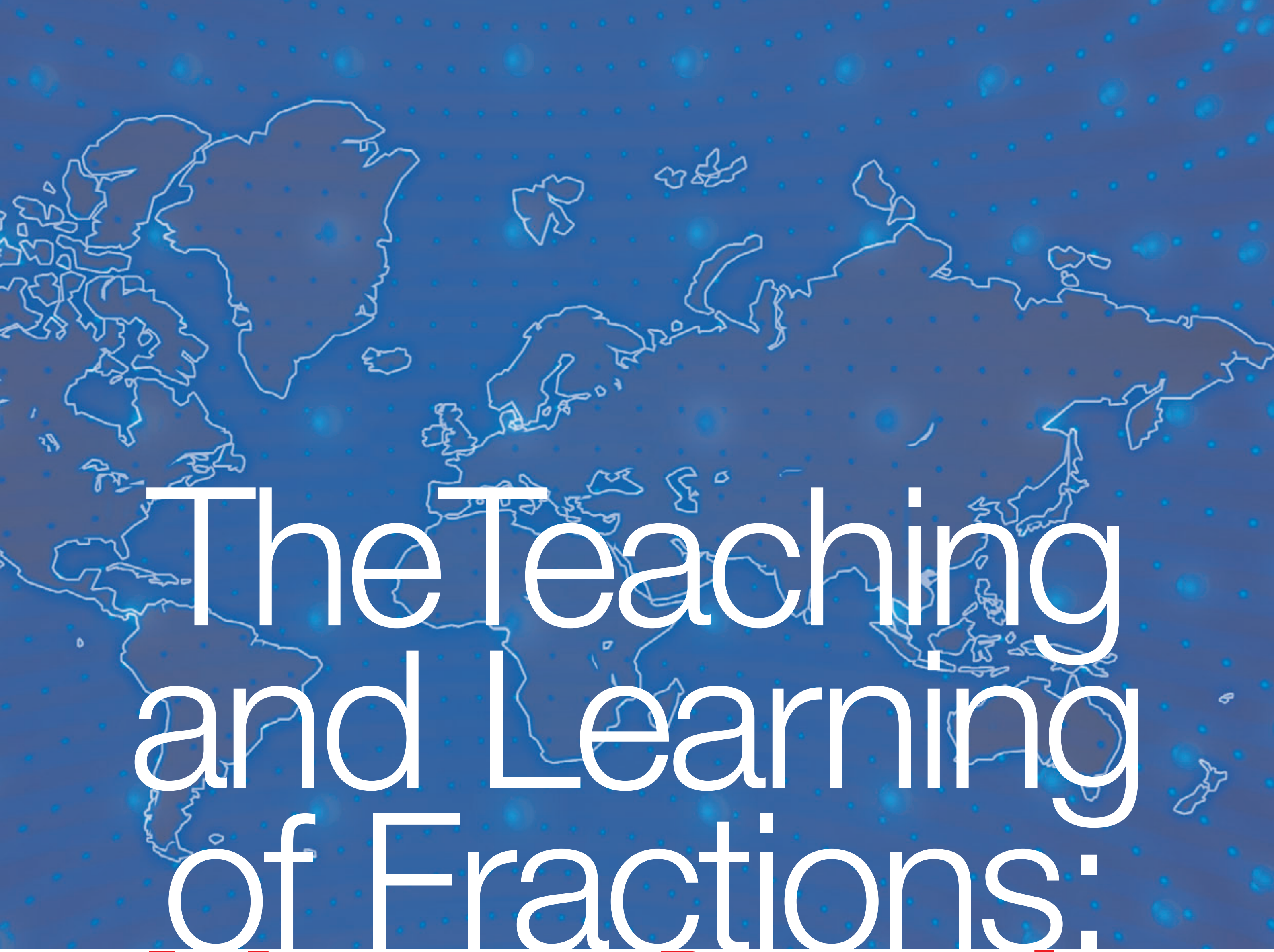

The Teaching and Learning of Fractions: A Japanese Perspective

The teacher posts a picture of two tree stumps on the blackboard (see **fig. 1**) and says, "I saw these two tree stumps at a park near my house yesterday. I was curious how big each stump was, so I wrapped a paper tape around each one so that I could measure them." Posting the first paper strip on the board (see **fig. 2**), she asks, "How much of a meter is the extra part?"

"We can use a decimal number!" a student says. The student goes up to the board to point out the equally spaced marks inside the one-meter segment of the paper strips. The teacher cuts off the extra piece.

"Can someone measure this length using his idea?" the teacher says.

Another student goes up to the board and places the extra piece underneath the one-meter segment. "It's 0.2 meters long," she says (see **fig. 3**).

The teacher cuts off the extra piece from the second paper strip and asks, "Can someone else measure this piece?"

A third student goes up to the board and places the extra piece underneath the one-meter segment. "Oh no, it doesn't match up," he says (see **fig. 4**).

"Move it a bit to your left," a student suggests. Another says, "No, you have to have the left end lined up."

"Hmm," the teacher says. "Maybe we can't use a decimal number to measure the length of this extra piece. Let's explore how we can express this length using a different idea."

By Tad Watanabe

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The vignette above describes how a Japanese teacher might begin the introductory unit on fractions in grade 4. In a previous article (Watanabe 2001), I suggested that formal instruction of fractions be delayed in the United States. In the same article, I also mentioned that fractions are not introduced until grade 4 in Japan. Japanese children consistently perform at or near the top in a variety of international studies, however, and their performance on fraction-related items is not an exception. This observation naturally raises the question "How are fractions taught in Japan?"

In this article, I will briefly discuss how fractions are treated in the Japanese national Course of Study (COS). Then I will present how Japanese elementary mathematics textbooks introduce fractions in grade 4. I will conclude by discussing some questions about how we teach fractions in the United States. The purpose of this article is not to judge the merits of the Japanese or U.S. approach to fraction instruction. Rather, it is my hope that this article will motivate an examination of our own practices by using the practices of another country as a lens to more clearly view our own fraction instruction.

Fractions in the Japanese Curriculum

Table 1 summarizes the treatment of fractions in the Japanese COS at different grades. Not only are fractions introduced later than in North America, but, in general, topics discussed at each grade level seem to be much more focused. In grade 4, when fractions are first introduced, the focus is on developing the meaning of fractions and mixed numbers. Comparison of fractions is limited to situations in which fractions have the same denominator. In grade 5, an intentional effort is made to expand the meaning of fractions to include the indicated division, e.g., 3/4 = 3 ÷ 4. During grade 5, discussions of fractions include the comparison of fractions with unlike denominators, but discussions of addition and subtraction are limited only to fractions with like denominators. A major emphasis in grade 5 is the development of the relationships among fractions, decimals, and whole numbers. Finally, in grade 6, arithmetic operations with fractions are investigated in depth.

In addition to the COS, the Japanese Ministry of Education also publishes a series of books titled *Commentary on the Course of Study* for each subject, which provide more detailed discussion and explanation of the COS. For example, according to

Table 1

From the Japanese course of study

Grade 4	Introduction of fractions; improper fractions and mixed numbers; comparison of fractions (with equal denominators)
Grade 5	Equivalent fractions; comparison of fractions (non-equal denominators); addition and subtraction of fractions with like denominators; fractions as quotient; relationships among fractions, decimals, and whole numbers
Grade 6	Addition and subtraction of fractions with unlike denominators; multiplication and division of fractions

Commentary on the Course of Study: Elementary School Mathematics (Ministry of Education 1999), five meanings of fractions exist. The fraction 2/3, for instance, means the following:

1. Two parts of a whole that is partitioned into three equal parts
2. Representation of measured quantities such as (2/3)l or (2/3)m
3. Two times of the unit obtained by partitioning 1 into 3 equal parts
4. A quotient fraction (2 ÷ 3)
5. A is 2/3 of B; if we consider B as 1 (a unit), then the relative size of A is 2/3.

The document goes on to say that in grade 4, when fractions are first introduced, the focus is on the first three meanings, whereas in grade 5, the meanings of fractions are expanded to include the fourth meaning. Grade 6 addresses fractions as proportions, the fifth meaning, when students study proportionality. Of course, teachers do not always refer to the COS or the elaboration document as they teach their everyday lessons. Rather, they use commercially published textbooks. So how are those textbooks organized as they introduce and develop fraction concepts? The next section focuses on how fractions are introduced and developed in grade 4.

Fractions in Japanese Textbooks

How fractions are introduced and developed

The following problem is the introductory problem in one of the two most widely used textbook series in Japan (Hosokawa et al. 2002):

Figure 1

Pictures of tree stumps posted by the classroom teacher

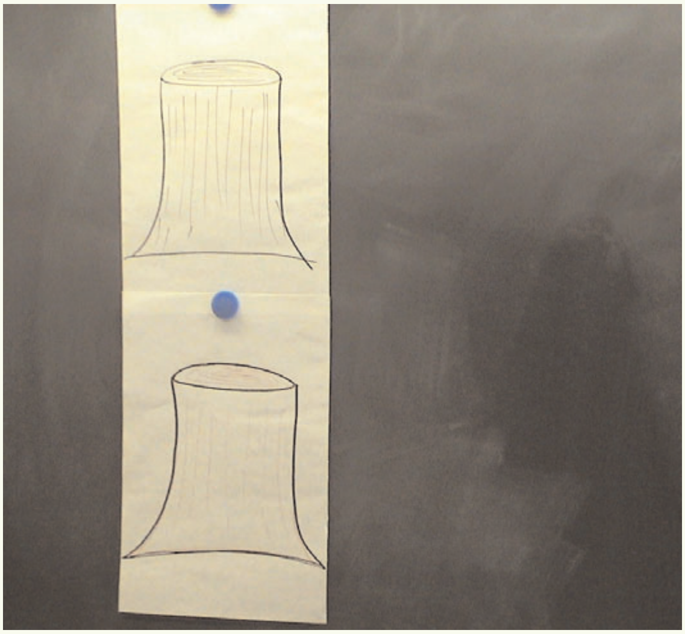
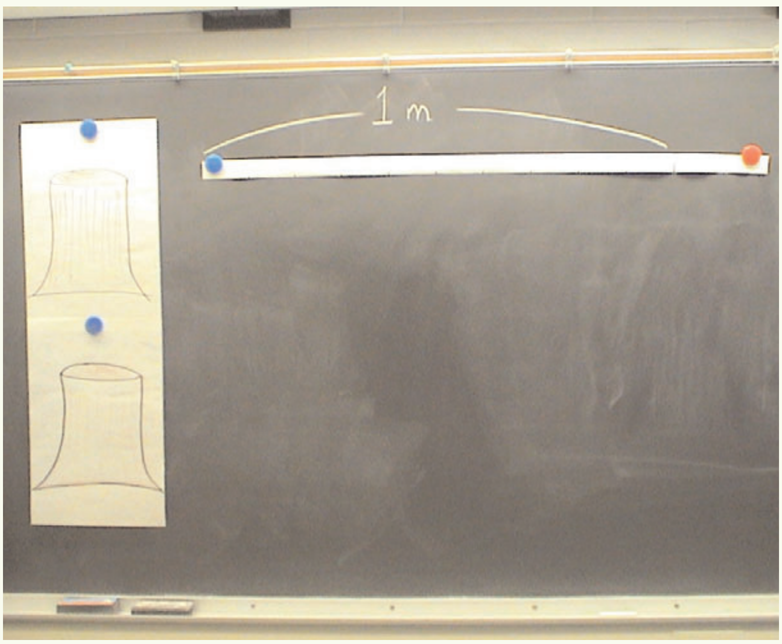


Figure 2

The teacher posts a paper strip for the circumference of the first stump.



> A teacher and two of her students measured the length of their arm spans. Each length was between 1 and 2 m. How can we express the extra part using a meter? [The problem includes a picture similar to **fig. 5**.]

This problem is very similar to that posed by the teacher in the opening vignette of this article, which came from the other most widely used textbook series (Sugiyama, Iitaka, and Ito 2002). Through problems such as these, the idea of unit fractions is developed. For example, this series states, "The extra piece for the teacher is the same length as one of the two equally partitioned lengths of 1 meter, and we say it is 1/2 of 1 meter."

After unit fractions are established, the textbook series introduces non-unit fractions less than one through a problem such as the following (Hosokawa et al. 2002):

> A paper tape that is one meter long has been partitioned into three equal pieces.
> a) How much of one meter is one piece?
> b) How much of one meter are two pieces together?

The textbook then provides answers: "The length that is 1/3 of 1 m ➔ 1/3 m," and "2 pieces of 1/3 m ➔ 2/3 m," respectively. Although some textbooks may discuss the part-whole interpretation of non-unit fractions first (i.e., (2/3)m is 2 of 3 equally partitioned lengths of 1 m), all textbook series emphasize this definition of a non-unit fraction (still as measured quantities) as a collection of unit fractions. Thus, for example, 3/4 meter is the length equal to three 1/4-meters. This notion is then extended to include fractions greater than or equal to one. Textbooks often include a figure such as **figure 6**. The final subunit of the grade 4 discussion of fractions includes mixed-number notations and comparison of fractions with like denominators.

Unique features

The above description of the grade 4 introduction and development of fractions reveals some unique features of the way in which fractions are taught in Japan. First, fractions are usually introduced and discussed in measurement contexts. The two introductory problems shown above are in the context of linear measurement. Another common measurement context involves liquid measures (liter). The primary reason for the Japanese textbooks' emphasis on measurement contexts is to help students

understand fractions as quantities. What we do not see much in the Japanese textbooks is the treatment of fractions as "*n* out of *m* parts." The teachers' manuals for the Japanese textbooks often discuss that the major challenge students face is to understand fractions as quantities, and the part-whole approach often contributes to this challenge. Thus, they introduce fractions as measured quantities less than one measurement unit.

With the emphasis on measurement rather than on the part-whole meaning of fractions, the Japanese textbooks treat non-unit fractions as a collection of unit fractions. Although this approach is usually discussed in commonly used elementary mathematics methods textbooks (e.g., Van de Walle 2004), it is rarely seen in our elementary mathematics textbooks, which primarily focus on the part-whole meaning of fraction. Thompson and Saldanha (2003) note that, with the part-whole meaning of fractions, students often have difficulty making sense of fractions greater than one.

Another point readers may have noticed is the lack of an area model that is so prevalent in textbooks published in the United States. Of the six currently available textbook series in Japan, only one uses the area model in the introduction of fractions. Even in that series, however, the initial problem is followed by a problem in the linear measure context. It is only in grades 5 and 6, when fractions are treated as numbers (not necessarily as measured quantities), that the textbooks start using area models. The only context presented in grade 4 that even comes close to an area model is the liquid measure context, which often includes a pictorial representation such as the one in **figure 7**.

Finally, another unique feature of the Japanese textbook series is the complete lack of the use of discrete models. As discussed above, the area model is not a common model in Japanese textbooks. However, when they discuss fractions in a liquid measure context, they often include a pictorial representation such as the one in **figure 8**. Although these models are not based on area measurement, readers can see a similarity. In contrast, none of the six textbook series uses a discrete model, that is, a fraction of a set of objects, in grade 4. In fact, the discussion of a fraction of a set of objects takes place in the context of the proportion-fraction, the fifth meaning of fractions discussed in the *Commentary*, which is not studied until grade 6.

Questions to Ponder

As I stated in the introduction, the purpose of this article is not to evaluate the Japanese or the North American approaches to teaching fractions. Rather, I hope this article will help us ask important questions about how we teach fractions to support student learning. This section shares some of the questions that I have begun to ask myself.

One of the unique features of Japanese textbooks is the explicit use of measurement contexts as they introduce fractions. Furthermore, the choice of the measurement context seems to be influenced by the type of model the authors want to use. Thus, in order to help students become fluent with the number line model of fractions, these textbooks use the linear model of fractions with pictures of measuring tapes. Furthermore, to make this pictorial representation "natural," some textbooks intentionally select the problem context involving the measures of paper tapes. Thus, the initial model is simply a pictorial representation of the actual items involved in the problem. The books go from an iconic representation to a more abstract representation (number line). That the dominant model in the U.S. textbook series is the area model seems clear. So what are the advantages of various fraction models? What are the potential limitations of these models? What challenges do students face as they try to make sense of and use any particular model? What are the

Figure 3

The extra piece of paper strip is 0.2 meter long.

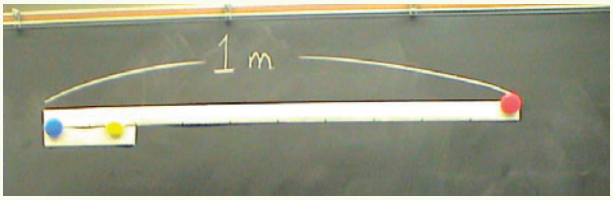


Figure 4

The extra piece of paper strip for the second tree stump would not match up with the decimal marks.

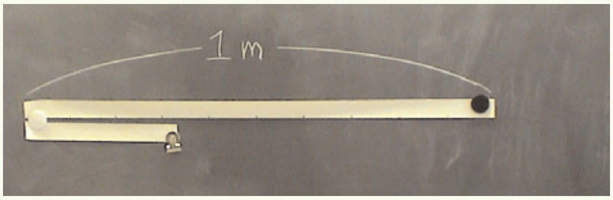
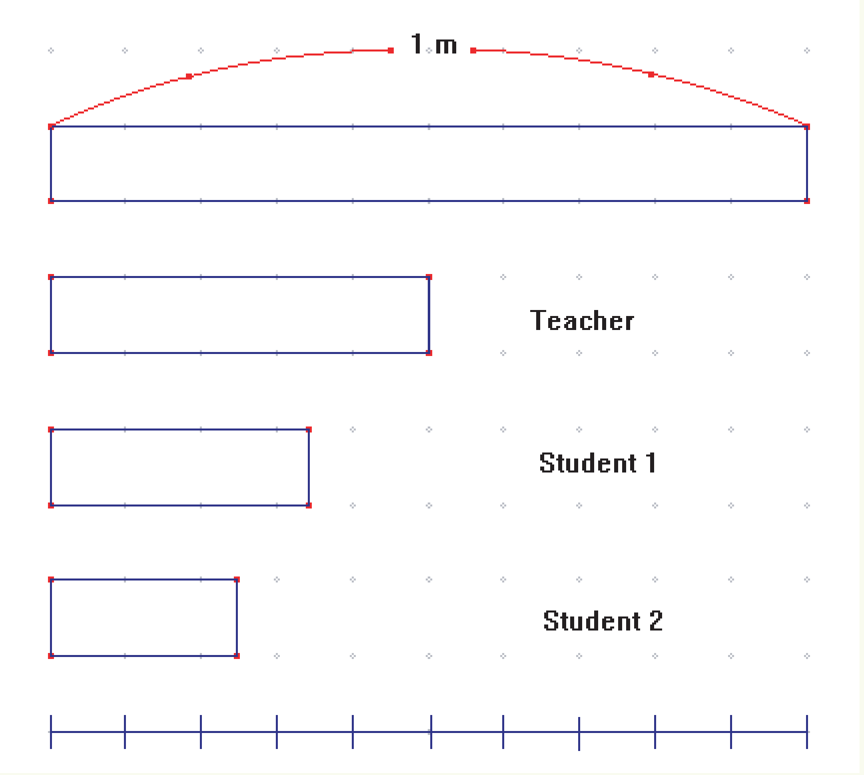


Figure 5

A picture such as this is included with the opening problem in a textbook. The segments for the teacher and the students show the segment only beyond 1 m.



advantages of using multiple models? What can we do to help students make better connections among various models?

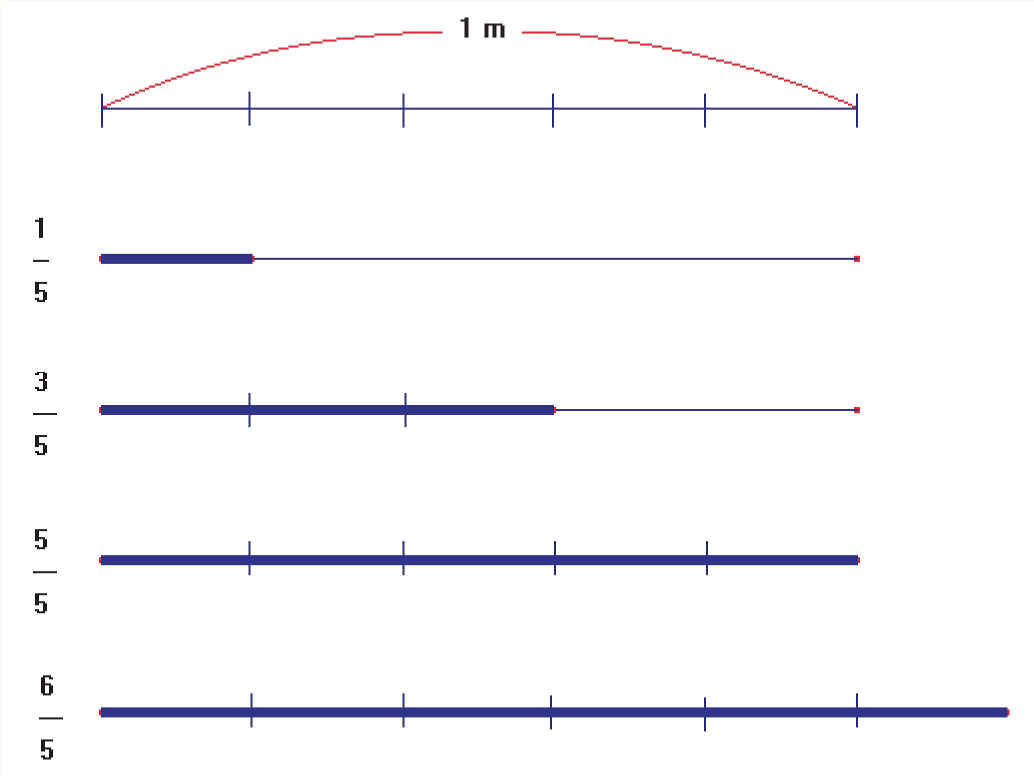
These questions are particularly relevant to U.S. classrooms because one of the reasons often cited for introducing fractions so early is that fractions are necessary in linear and liquid measurement contexts while using the customary units. Therefore, it seems reasonable to make much more explicit connections between the study of fractions and the study of linear and liquid measurements. However, this is not the case in most of the textbooks today. Perhaps introducing fractions in a context similar to what is often seen in the Japanese textbooks might be helpful for children to make sense of both fractions and measurement with a fraction of a unit.

Another uniqueness of the Japanese approach is how they consider non-unit fractions as a collection of unit fractions. This is an approach we rarely see in any available curriculum in the United States (Thompson and Saldanha 2003). What is the advantage of such an approach at the introductory stage? What are the advantages of the part-whole approach, which seems to dominate our fraction instruction? How can we help students make connections among these various meanings of fractions? Are we intentionally helping students make such connections? What models or problem contexts might be useful in helping students make those connections?

Finally, the Japanese approach seems to show that, through a focused and coherent approach, it is possible to introduce fractions much later than is typically done in North America and to reach the same goal by the end of grade 6. So what are the advantages of introducing fractions at an earlier age? What special challenges may students face because fractions are introduced early? Is any specific fraction model or problem context more appropriate when fractions are introduced earlier, before students study other related ideas such as area measurement or multiplication and division operations?

Figure 6

Textbooks include a diagram such as this to show how non-unit fractions are composed of unit fractions.



I believe that reflecting critically on our own practices and grappling with these and other questions will help us improve the teaching and learning of fractions.

Concluding Comments

This article described only how the Japanese curriculum and textbooks treat fractions. However, we all agree that mathematical ideas do not exist in isolation. Fractions are no exception. Many ideas are relevant to fractions—multiplication and division operations, measurement, composing and decomposing of geometric figures, and so on. Some of the unique features in the Japanese curriculum make more sense when we consider how those related concepts are treated in the Japanese curriculum. Therefore, we should be careful, as we grapple with the questions raised above (and others), that we keep our eyes on a bigger picture. We cannot simply change our approach to the teaching and learning of fractions.

Figure 7

Pictorial representation of 4/5 liter

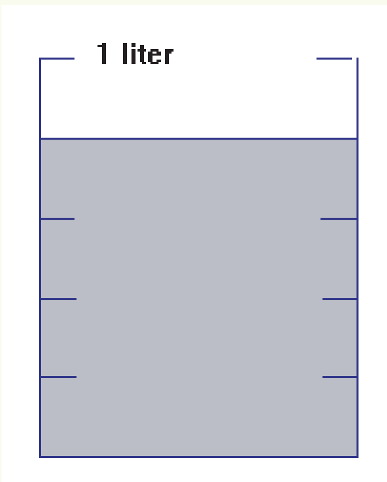
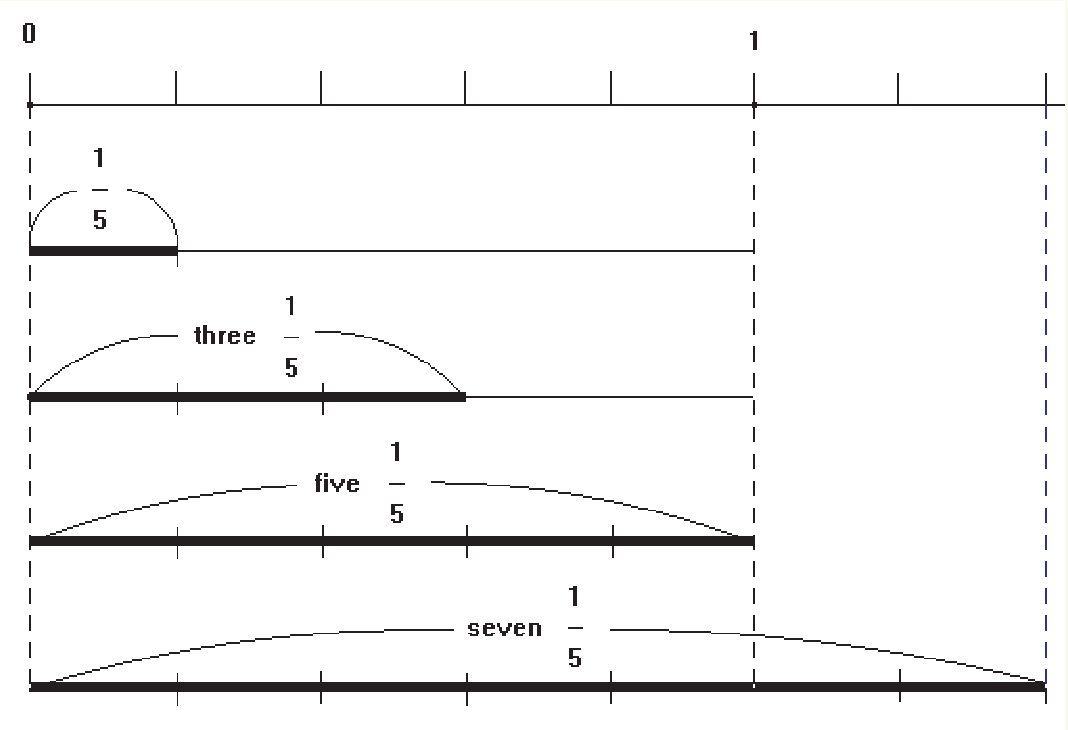


Figure 8

Connecting fraction models to the number line



We must make sure that our curriculum is cohesive and coherent by examining, systematically, the way in which we teach all our mathematics concepts.

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Editor's Note

Teaching Children Mathematics has published a number of articles on the topic of teaching fractions in the elementary school. See the sampling below:

Olive, John. "Bridging the Gap: Using Interactive Computer Tools to Build Fraction Schemes." *Teaching Children Mathematics* 8 (February 2002): 356–61.

Ploger, Don, and Michael Rooney. "Teaching Fractions: Rules and Reasons." *Teaching Children Mathematics* 12 (August 2005): 13–17.

Riddle, Margaret, and Bette Rodzwell. "Fractions: What Happens between Kindergarten and the Army?" *Teaching Children Mathematics* 7 (December 2000): 202–6.

Watanabe, Tad. "Representations in Teaching and Learning Fractions." *Teaching Children Mathematics* 8 (April 2002): 457–63.

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